

## P-term potentials from 4-D supergravity

---

**Clare Burrage**

*Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences*

*Cambridge CB2 0WA, United Kingdom*

*E-mail: C.Burrage@damtp.cam.ac.uk*

**Anne-Christine Davis**

*Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences*

*Cambridge CB2 0WA, United Kingdom*

*E-mail: A.C.Davis@damtp.cam.ac.uk*

**ABSTRACT:** P-term inflation arises in some models of brane inflation. Within  $N=2$  supersymmetry the scalar potential contains a vector of Fayet-Iliopoulos (FI) terms  $\xi_i$ . Depending on the direction of this vector it is possible to get D-term and F-term inflation or a mix of these models. In this paper we review the problems of embedding the P-term model in supergravity and show how these can be solved by considering the truncation from an  $N=2$  theory to  $N=1$ . We show that with a simple gauging the scalar potential can include F-term or D-term parts but not both. The gauging can be altered so that both F-terms and D-terms containing FI constants can be included. In all cases we display the inflationary trajectory and, if it exists, the supersymmetric minimum.

**KEYWORDS:** Supergravity Models, Supersymmetry Breaking, Extended Supersymmetry.

---

## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. P-term models in supersymmetry and rigid supergravity</b>	<b>2</b>
2.1 Cosmology	4
<b>3. P-term models in supergravity</b>	<b>5</b>
3.1 Reduction from $N = 2$ to $N = 1$ supergravity	6
3.2 Building a P-term potential	9
3.2.1 Gauging one isometry	10
3.2.2 Gauging two isometries	13
3.2.3 Gauging three isometries	14
<b>4. Conclusion</b>	<b>15</b>
<b>A. Euler angles</b>	<b>16</b>
<b>B. The effective potential</b>	<b>17</b>

---

## 1. Introduction

P-term inflation is a type of hybrid inflation [1] that appears as the effective theory of some brane inflation models, in particular the D3/D7 version [2, 3] of the KKLMST scenario [4, 5]. In the framework of  $N=2$  supersymmetric gauge theory a P-term model was introduced in [6], with a global  $SU(2,2|2)$  superconformal gauge theory that corresponds to a dual gauge theory of supersymmetric D3/D7 branes [7]. Superconformal  $SU(2,2|2)$  symmetry can be broken down to  $N=2$  supersymmetry by the vev of the auxiliary triplet field  $P_i$  of the vector multiplet. The P-term model in  $N=2$  supersymmetry contains a triplet of FI terms  $\xi_i$  which arise from a magnetic flux triplet in the D3/D7 brane construction. Truncating one of the supersymmetries leaves an  $N=1$  model, which describes both F- and D-term potentials and also potentials which are a mix of the two.

The status of the FI terms becomes more complicated when the supersymmetry is made local. In [8, 9] a supergravity was constructed with a locally supersymmetric extension of the FI term of the Abelian vector multiplet. Local SUSY with an FI term requires axial gauging of the gravitino and the gaugino, known as local R-symmetry. It has long been known [10] that a  $U(1)$  gauge theory with an FI term can be coupled to supergravity in a way that preserves the gauge invariance only if the superpotential of the theory is invariant under the R symmetry and charged under the  $U(1)$  symmetry. The behaviour of  $N=1$  supergravity with FI terms and an R-symmetry was studied in [11] using

the superconformal approach to supergravity [12]. In the superconformal picture gauge symmetries commute with local superconformal symmetries, which is shown to imply that in supergravity a superpotential cannot be gauge invariant. This has consequences for the construction of P-term potentials in supergravity as the charges of the scalar fields differ in supergravity from their supersymmetric values, as will be discussed in section 2. A straightforward generalisation of the supersymmetric formalism is not possible; to study P-term inflation it is necessary to construct a P-term potential in supergravity.

In a supersymmetric P-term model we start with two supersymmetries and an  $SO(3)$  choice of direction for the FI vector. One of the symmetries is then broken to leave an  $N=1$  SUSY theory with a P-term potential. We wish to repeat this procedure in supergravity; to construct a P-term model in the  $N=2$  theory and break one of the symmetries to leave an  $N=1$  P-term theory. However until recently it was not known how to obtain FI terms in  $N=2$  supergravity, embedding the P-term model into supergravity was not thought to be possible. In [13] a model was constructed which had FI constants in the D-term of an  $N=2$  theory. Section 3 describes a generalisation of this construction in which FI terms are also produced in the superpotential giving rise to a P-term supergravity potential.

The paper is organised as follows. Section 2 reviews the supersymmetric version of the P-term model in flat and curved spacetime. A heuristic approach to the coupling to gravity is taken [7] whereby one of the supersymmetries is made local and the resulting theory is treated as  $N=1$  supergravity. The problem of how to embed P-term inflation into supergravity is addressed in section 3. Section 3.1 reviews the way an  $N=2$  supergravity theory is truncated to an  $N=1$  theory and the conditions this imposes on the content of the theory. Section 3.2 gives the details of the geometry we use to construct a P-term potential. We show how a particular choice of Killing vector for the quaternionic geometry can give rise to FI terms, however in section 3.2.1 we show that gauging only one isometry means that D-terms or F-terms are allowed in the reduced potential, but not both. Sections 3.2.2 and 3.2.3 show how more complicated gaugings can give rise to FI terms in the superpotential and D-term of the theory and we construct the resultant P-term potential. For each gauging we consider the vacuum structure of the reduced scalar potential and discuss its usefulness for cosmology. We conclude in section 4.

## 2. P-term models in supersymmetry and rigid supergravity

We begin with a review of the P-term potential in supersymmetry. Take an  $N=2$  supersymmetric theory which contains one hypermultiplet and one vector multiplet and arrange the two charged scalar fields into a multiplet of charge +1

$$h = \begin{pmatrix} \phi_+ \\ \phi_-^* \end{pmatrix} \tag{2.1}$$

so that the P-term potential for bosonic scalars can be written

$$V = \frac{g^2}{8} \sum_{i=1}^3 (h^\dagger \sigma_i h - \xi_i)^2 + \frac{g^2}{2} |\phi_0|^2 h^\dagger h \tag{2.2}$$

This potential was constructed in [7].  $\phi_0$  is the scalar in the uncharged chiral multiplet,  $g$  is a gauge coupling constant,  $\sigma_i$  are the Pauli matrices and  $\xi_i$  is a constant Fayet-Iliopoulos 3-vector. If  $\xi_1 = \xi_2 = 0$  this potential is the super-Bogomol'nyi limit<sup>1</sup> of a D-term potential; if  $\xi_2 = \xi_3 = 0$  this is the Bogomol'nyi limit of an F-term potential. We split the three vector  $\xi_i$  into the product of a rotational part described by a SO(3) matrix  $R$  and a magnitudinal part  $(0, 0, \xi)$

$$\xi_i = (R^{-1})_{ij} \delta_{j3} \xi \tag{2.3}$$

If  $R_{ij} = \delta_{ij}$  then (2.2) is a D-term potential, so all P-term potentials are described by rotations from the D-term case. The class of P-term potentials is parameterised by two Euler angles,  $\psi$ ,  $\theta$ , and  $\xi$ , the magnitude of the FI vector. Our conventions for Euler angles are given in appendix A.

The scalar potential is constructed from the superpotential and D-term as  $V = |\partial W|^2 + \frac{g^2}{2} D^2$  with

$$W = \frac{g\phi_0}{2\sqrt{2}}(P_1 - iP_2) \tag{2.4}$$

$$D = \frac{1}{2}P_3 \tag{2.5}$$

where

$$P_i = h^\dagger \sigma_i h - (R^{-1})_{i3} \xi \tag{2.6}$$

So a P-term potential contains constant FI terms in the superpotential and the D-term.

To look at inflation in a P-term theory we need to include the gravitational sector. One way to proceed [7] is to couple the N=2 supersymmetric theory to N=1 supergravity by choosing a supersymmetry, making it local, and treating it as if it were the only one. As discussed in the introduction if a U(1) gauge theory with an FI term is to be consistently coupled to supergravity in a way that preserves gauge invariance the superpotential must be invariant under the R-symmetry but transform under the U(1) symmetry. This alters the charges of the fields appearing in the superpotential. In supersymmetry the scalar fields  $\phi_\pm$ ,  $\phi_0$  have charges  $Q_\pm = \pm 1$ ,  $Q_0 = 0$ , and the superpotential is uncharged. As an example of how this changes in supergravity take  $W \propto \phi_0 \phi_+ \phi_-$  and an FI constant in the D-term. Then the fields have charges

$$q_i = Q_i - \rho_i \frac{\xi}{M_P^2}, \quad \sum \rho_i = 1 \tag{2.7}$$

As  $M_P \rightarrow \infty$  we regain  $q_i = Q_i$ , indeed with a generic choice of superpotential the charges always return to their supersymmetric values in this limit.  $M_P \rightarrow \infty$  is known as the rigid limit of supergravity [11] and describes supersymmetry in a curved space-time. Unless the rigid limit is being considered it no longer makes sense to combine  $\phi_+$ ,  $\phi_-^*$  into the multiplet (2.1), so except in this limit it is unclear how to proceed with this formalism in supergravity.

---

<sup>1</sup>as defined in [14]

## 2.1 Cosmology

To find the form of the scalar potential in rigid supergravity take the minimal Kähler potential

$$K = \frac{|\phi_+|^2 + |\phi_-|^2 + |\phi_0|^2}{M_P^2} \quad (2.8)$$

minimal vector kinetic terms, and the superpotential (2.4), and D-term (2.5). Note that the  $M_P \rightarrow \infty$  limit should not be taken at this stage but at the end of the calculation. Compute the scalar potential in the usual way

$$\begin{aligned} V &= e^K \left\{ \left| \frac{\partial W}{\partial \phi_i} + \frac{\phi_i^* W}{M_P^2} \right|^2 - 3 \left| \frac{W}{M_P} \right|^2 \right\} + \frac{g^2}{2} D^2 \quad (2.9) \\ &= \frac{g^2}{2} \left\{ |\phi_+ \phi_-|^2 + |\phi_0 \phi_-|^2 + |\phi_0 \phi_+|^2 \right. \\ &\quad \left. - \sin \theta \sin \psi \xi (\phi_+ \phi_- + \bar{\phi}_+ \bar{\phi}_-) + (\sin \theta \sin \psi \xi)^2 \right\} + \mathcal{O} \left( \frac{|\phi_i|^2}{M_P^2} \right) \\ &\quad + \frac{g^2}{8} \left( |\phi_+|^2 - |\phi_-|^2 - \cos \theta \xi \right)^2 \quad (2.10) \end{aligned}$$

The term in braces in (2.9) is the F-term part of the potential, the second term is the D-term. The full expression for the potential to all orders in  $|\phi_i|/M_P$  is given in appendix B. This potential has two types of extrema:

**Inflationary valley.**  $\phi_+ = \phi_- = 0$  is a stationary point of the potential in the  $\phi_{\pm}$  directions. The extremum is a minimum if

$$2|\phi_0|^2 > \xi \quad (2.11)$$

and a maximum otherwise. Inflation occurs when the fields are rolling in this valley and ends when the critical point is reached.

The effective potential during inflation, when  $\phi_+ = \phi_- = 0$ , is

$$V = \frac{\xi^2 g^2}{2} \left\{ 1 + \sin^2 \theta \left[ \frac{1}{2} \frac{|\phi_0|^4}{M_P^4} + \mathcal{O} \left( \frac{|\phi_0|^5}{M_P^5} \right) \right] \right\} \quad (2.12)$$

P-term inflationary potentials are parameterised by  $0 \leq \sin^2 \theta \leq 1$ . The two limiting cases are  $\sin \theta = 0$  which gives the D-term potential and  $\sin^2 \theta = 1$  which gives the F-term potential. In the rigid limit the inflationary potential is flat for all P-term potentials. In the D-term case all corrections to the flat potential vanish which suggests that the D-term inflationary potential remains flat in supergravity unlike all other P-term potentials.

**Supersymmetric minimum.** The supersymmetric minimum is the point at which

$$\frac{\partial W}{\partial \phi_i} + \frac{\phi_i^* W}{M_P^2} = 0 \quad (2.13)$$

$$D = 0 \quad (2.14)$$

This occurs at

$$\begin{aligned}
 \phi_0 &= 0, \\
 |\phi_+|^2 - |\phi_-|^2 &= \xi \cos \theta, \\
 e^{i\psi} \cos\left(\frac{\theta}{2}\right) \phi_- &= i \sin\left(\frac{\theta}{2}\right) \bar{\phi}_+
 \end{aligned}
 \tag{2.15}$$

where the vacuum manifold is a circle. After the phase transition at the end of inflation the charged fields waterfall into the supersymmetric minimum. Cosmic strings may be formed by the Kibble mechanism [15].

This is the vacuum structure required for hybrid inflation. All of the P-term models described here give rise to hybrid inflation and allow cosmic string formation at the end of inflation.

### 3. P-term models in supergravity

The analysis of section 2, which describes supersymmetry in a curved space-time gives an indication of the behaviour of a P-term model in supergravity. However for a complete analysis we need to construct a P-term potential in supergravity. This means finding an N=2 supergravity which can be truncated to N=1 in such a way as to give FI terms in the superpotential and D-term of the theory. As in the supersymmetric version of the theory we start with an N=2 supergravity theory containing a constant FI term in each of the three components of the moment map and an arbitrary choice of direction. We then truncate one of the supersymmetries to get an N=1 supergravity theory containing FI terms. For notational convenience from now on we will use units in which  $M_P = 1$ .

Consider an N=2 supergravity theory [16] with  $n_H$  hyper-multiplets,  $n_V$  vector multiplets, and  $n_C$  chiral multiplets. The hyper-multiplet sector describes a quaternionic-Kähler manifold which has SU(2) connection  $\omega^x$ , and complex structures  $J_{uv}^x$ , where  $x = 1, 2, 3$  and  $u = 1, \dots, 4n_H$ . The vector multiplet sector describes a special-Kähler Manifold with a  $2(n_V + 1)$  component symplectic section

$$V = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} = e^{\frac{K}{2}} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}
 \tag{3.1}$$

$\Lambda = 0, 1, \dots, n_V$ .  $K$  is the Kähler potential

$$e^{-K(z, \bar{z})} = -iX^\Lambda \bar{F}_\Lambda + iF_\Lambda \bar{X}^\Lambda
 \tag{3.2}$$

where  $z^i$  are the coordinates on the manifold. The symplectic section may be written in terms of a prepotential  $F(X^\Lambda)$  so that  $F_\Lambda = \partial F / \partial X^\Lambda$ .

Historically the problem with lifting P-term inflation to supergravity was that it was not known how to get constant FI terms in N=2 supergravity. In N=2 supergravity the moment map which corresponds to the Killing vector  $k_\Lambda$  is given by

$$4n_H P_\Lambda^x = -J^{xuv} \partial_u k_{v\Lambda}
 \tag{3.3}$$

Constant terms in the moment maps of the N=2 theory give rise to FI terms. Except in a few extremal cases (for example  $n_H = 0$ , or the rigid limit when the SU(2) curvature vanishes) it was not known how to add an arbitrary constant to  $P_\Lambda$  and still satisfy (3.3). Recently in [13] a moment map was constructed in N=2 supergravity which gave a D-term potential containing a constant FI term. This was done by considering the truncation to an N=1 theory. (In [17] this was extended to admit an axion-dilaton field). The moment maps were written as [18]

$$P_\Lambda^x = \frac{1}{2}r_\Lambda^x + \iota_\Lambda \omega^x \tag{3.4}$$

where  $r_\Lambda^x$  is the SU(2) compensator. A Killing vector only preserves  $\omega^x$  and  $J^x$  up to an SU(2) transformation, and the compensator is defined so that the following equations are satisfied

$$\mathcal{L}_\Lambda \omega^x = \frac{-1}{2} \nabla r_\Lambda^x \tag{3.5}$$

$$\mathcal{L}_\Lambda J^x = \epsilon^{xyz} r_\Lambda^y J^z \tag{3.6}$$

where  $\mathcal{L}_\Lambda$  denotes the Lie derivative with respect to  $k_\Lambda$ . In some manifolds it is possible to choose a Killing vector so that the compensator is constant, resulting in a constant term in the moment map. In section 3.2 we show how a particular choice of Killing vectors can lead to constant terms in the superpotential and the D-term of the reduced theory and thus to a P-term potential.

### 3.1 Reduction from $N = 2$ to $N = 1$ supergravity

The relationship between different P-term models is a result of the symmetries of the N=2 theory so we begin our construction of a supergravity P-term model in N=2 supergravity. The only known way to get FI terms in N=2 supergravity is by truncating the theory to one which resembles an N=1 supergravity containing FI terms [13] where we can construct a useful model of inflation. This parallels the discussion of P-term inflation in [7] where the P-term model was constructed by taking an N=2 supersymmetry and then making one of the symmetries local and disregarding the other to move to N=1 supergravity.

The constraints imposed by the truncation from N=2 to N=1 supergravity are known [19, 20] and we review them here. The truncation is done by constraining the theory so that the N=2 supergravity transformations look like those of an N=1 theory. Additional conditions come from ensuring that the reduced theory is consistent. A full derivation of the truncation conditions is not necessary for this work; we state here the conditions which constrain the moment maps or the Killing vectors of the N=2 theory. This will allow us to compute the reduced scalar potential.

The reduction truncates the spin 3/2 multiplet as the second gravitino,  $\psi_{2\mu}$ , and the graviphoton are not present in N=1 supergravity. The N=2 gravitino transformations are

$$\delta\psi_{A\mu} = \hat{\nabla}_\mu \epsilon_A + (igS_{AB}\eta_{\mu\nu} + \epsilon_{AB}T_{\mu\nu}^-)\gamma^\nu \epsilon^B \tag{3.7}$$

where

$$\hat{\nabla}_\mu \epsilon_A = \mathcal{D}_\mu \epsilon_A + \hat{\omega}_{\mu|A}^B \epsilon_B + \hat{Q}_\mu \epsilon_A \tag{3.8}$$

The SU(2) gauge connection is

$$\hat{\omega}_A^B = \omega_A^B + g_{(\Lambda)} A^\Lambda P_\Lambda^x (\sigma^x)_A^B \quad (3.9)$$

and the U(1) gauge connection is

$$\hat{Q} = Q + g_{(\Lambda)} A^\Lambda P_\Lambda^0 \quad (3.10)$$

where  $\omega_A^B$  and  $Q$  are the connections of the ungauged theory.  $P_\Lambda^0$  is the holomorphic momentum map, the index 0 is to distinguish it from the triholomorphic momentum map  $P_\Lambda^x$ .  $S_{AB}$  describes the shift of the gravitino

$$S_{AB} \equiv \frac{i}{2} P_\Lambda^x \sigma_{AB}^x L^\Lambda \quad (3.11)$$

The ‘dressed’ graviphoton is

$$T_{\mu\nu}^- \equiv 2i \text{Im} \mathcal{N}_{\Lambda\Sigma} L^\Sigma F_{\mu\nu}^{\Lambda-} \quad (3.12)$$

where  $\mathcal{N}_{\Lambda\Sigma}$  is the N=2 vector kinetic matrix which can be computed from the prepotential  $F(X^\Lambda)$  as

$$\mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + \frac{i N_{\Lambda\Delta} N_{\Sigma\Gamma} X^\Delta X^\Gamma}{N_{\Theta\Upsilon} X^\Theta X^\Upsilon} \quad (3.13)$$

with  $F_{\Lambda\Sigma} = \partial^2 F(X^\Delta) / \partial X^\Lambda \partial X^\Sigma$  and  $N_{\Lambda\Sigma} = 2 \text{Im} F_{\Lambda\Sigma}$ .

For the truncation to an N=1 theory to be consistent the transformation of the second gravitino,  $\delta\psi_{2\mu}$ , must be set to zero. In the ungauged case this means

$$T_{\mu\nu}^- = 0 \quad (3.14)$$

The most general way to find solutions to (3.14) is to view it as an orthogonality relation between the vector and scalar sectors. To satisfy this relation we split the index  $\Lambda = \{\Lambda, X\}$  so that  $\Lambda = 1, \dots, n'_V = n_V - n_C$  and runs over the retained vectors of the reduced theory and  $X = 0, \dots, n_C$  and runs over the retained scalars. To satisfy (3.14) set  $F_{\mu\nu}^X = 0$  and  $\text{Im} \mathcal{N}_{\Lambda\Sigma} L^\Sigma = 0$  which requires

$$L^\Lambda = 0 \quad (3.15)$$

In a gauged theory the vanishing of the second gravitino gives the additional constraints  $S_{21} = 0$  and  $\hat{\omega}_1^2 = 0$ . These are satisfied if

$$P_\Lambda^1 = P_\Lambda^2 = 0 \quad (3.16)$$

In an N=1 theory the gravitino transformation is

$$\delta\psi_\mu = D_\mu \epsilon + \hat{Q}_\mu \epsilon + iL(z, \bar{z}) \gamma_\mu \epsilon \quad (3.17)$$

where

$$L(z, \bar{z}) = W(z) e^{\frac{1}{2} K(z, \bar{z})} \quad (3.18)$$



comparing this with the reduction of (3.7) shows that

$$\begin{aligned} L(z, \bar{z}) &= \frac{i}{2} g_{(\Lambda)} L^\Lambda (P_\Lambda^1 - iP_\Lambda^2) \\ &= \frac{i}{2} g_{(X)} L^X (P_X^1 - iP_X^2) \end{aligned} \tag{3.19}$$

by (3.15).

The reduction of the transformations in the hypermultiplet and vector multiplet sectors should be considered separately. For the truncated theory to have the correct form for N=1 supergravity the quaternionic manifold of the hyper-multiplet sector must be reduced to a Kähler-Hodge one and the special-Kähler manifold of the vector sector reduced to a smaller Kähler manifold. Reducing the vector multiplet sector requires the truncation of the gaugino transformations. The N=1 gaugino transformation is

$$\delta\lambda^\Lambda = \mathcal{F}_{\mu\nu}^{-\Lambda} \gamma^{\mu\nu} \epsilon + iD^\Lambda \epsilon \tag{3.20}$$

For this to be the reduction of the N=2 theory

$$D^\Lambda \equiv (\text{Im}\mathcal{N}^{-1})^{\Lambda\Sigma} (P_\Sigma^0 + P_\Sigma^3) \tag{3.21}$$

where  $P_\Sigma^0$  is given by

$$P_\Sigma^0 = 2i \text{Im}\mathcal{N}_{\Sigma\Lambda} f_{XY}^\Lambda \bar{L}^X L^Y \tag{3.22}$$

and gives the special-Kähler manifold contribution to the D-term.  $f_{\Delta\Sigma}^\Lambda$  are the structure constants of the N=2 gauge group.  $P_\Lambda^0$  vanishes for an Abelian gauging. For the truncation of the gaugino to be consistent we must ensure

$$P_X^3 = 0, \quad P_X^0 = 0 \tag{3.23}$$

The truncation also requires that the vector kinetic matrix,  $f_{\Lambda\Sigma}$ , of the reduced theory is

$$f_{\Lambda\Sigma} = 2\tilde{\mathcal{N}}_{\Lambda\Sigma} \tag{3.24}$$

Truncation of the hypermultiplet sector requires the holonomy of the quaternionic manifold to be reduced because in N=1 supergravity all the scalars must lie in chiral multiplets with Kähler-Hodge (K-H) structure. This is equivalent to selecting an  $n_H$  dimensional complex submanifold on which the  $2n_H$  extra degrees of freedom are frozen. Let  $q^u$  be the coordinates on the quaternionic manifold, the reduction requires  $q^{4s+3} = q^{4s+4} = 0$ ,  $s = 0, \dots, n_H - 1$ . Then  $w^s = q^{1+4s} + iq^{2+4s}$  are the  $n_H$  holomorphic coordinates on the K-H manifold. We define  $n^t$  as the real coordinates along directions orthogonal to the reduced K-H manifold. This decomposition of indices allows us to write down the last set of conditions needed for the truncation; by considering the gauge groups in the original and truncated theories it can be shown that the Killing vectors of the quaternionic manifold are required to satisfy

$$k_X^s = 0, \quad k_\Lambda^t = 0 \tag{3.25}$$

For clarity we will list here all the truncation conditions needed to construct the scalar potential

$$L^\Lambda = 0 \tag{3.26}$$

$$P_\Lambda^1 = P_\Lambda^2 = 0 \tag{3.27}$$

$$P_X^3 = P_X^0 = 0 \tag{3.28}$$

$$k_X^s = k_\Lambda^t = 0 \tag{3.29}$$

Note that the moment maps that generate the superpotential have a scalar index and the moment maps that generate the D-term have a vector index.

To compute the P-term potential we start with the N=2 scalar potential

$$V_{N=2} = \bar{L}^\Lambda L^\Sigma \left( g_{\mathcal{I}\bar{\mathcal{J}}} k_\Lambda^{\mathcal{I}} k_\Sigma^{\bar{\mathcal{J}}} + 4h_{uv} k_\Lambda^u k_\Sigma^v \right) + P_\Lambda^x P_\Sigma^x \left( \frac{-1}{2} (\text{Im}\mathcal{N}^{-1})^{\Lambda\Sigma} - \bar{L}^\Lambda L^\Sigma \right) - 3P_\Lambda^x P_\Sigma^x \bar{L}^\Lambda L^\Sigma \tag{3.30}$$

where  $h_{uv}$  is the metric on the quaternionic manifold, and  $\mathcal{I} = 1, \dots, n_V$  is the world index. After truncation this becomes

$$V_{N=1} = 4 \left( -3L\bar{L} + g^{i\bar{j}} \nabla_i L \nabla_{\bar{j}} \bar{L} + g^{s\bar{s}} \nabla_s L \nabla_{\bar{s}} \bar{L} + \frac{1}{16} \text{Im}f_{\Lambda\Sigma} D^\Lambda D^\Sigma \right) \tag{3.31}$$

which can be computed from (3.13), (3.19) and (3.21) if the moment maps and prepotentials are known.

### 3.2 Building a P-term potential

We choose the simplest geometries for the vector and hypermultiplet sectors which contain enough degrees of freedom to allow a scalar matter field and an inflaton after the truncation. The matter field is found in the hypermultiplet sector where we use the standard quaternionic geometry  $\frac{\text{Sp}(1,1)}{\text{Sp}(1)\text{Sp}(1)}$  with metric

$$ds^2 = dh^2 + e^{-2h} (db_1^2 + db_2^2 + db_3^2) \tag{3.32}$$

and connections

$$\omega^x = \frac{-1}{2} e^{-h} db^x \tag{3.33}$$

If  $n_H = 1$  the truncation of this is a Kähler-Hodge manifold which depends on only one complex scalar field. This will be the matter field. The inflaton is found in the vector sector of the N=2 theory. We take  $n_V = 2$  so that the special-Kähler manifold contains a six component symplectic section. We choose the prepotential

$$F(X) = \frac{-i}{2} ((X^0)^2 - (X^1)^2 - (X^2)^2) \tag{3.34}$$

which gives the minimal special geometry

$$X^\Lambda = \begin{pmatrix} 1 \\ z_1 \\ z_2 \end{pmatrix}, \quad F_\Lambda = \begin{pmatrix} -iX^0 \\ iX^1 \\ iX^2 \end{pmatrix} = \begin{pmatrix} -i \\ iz_1 \\ iz_2 \end{pmatrix} \tag{3.35}$$

with  $n_C = 1$  so that  $X = 0, 1$  and  $\Lambda = 2$  a consistent truncation requires  $L^\Lambda = 0$  (3.26), hence  $X^2 = z_2 = 0$ . This leaves the scalar field  $z_1$  to be the inflaton which from now on we write as  $z$ . Note that smaller special-Kähler manifolds do not have enough scalar degrees of freedom to give an inflaton after truncation. The Kähler potential (3.2) of this geometry is

$$K_V = -\ln(2(1 - z\bar{z})) \tag{3.36}$$

$0 \leq |z| < 1$ . The vector kinetic matrix (3.13) can be computed from the prepotential (3.34) as

$$\mathcal{N}_{\Lambda\Sigma} = \frac{i}{(z^2 - 1)} \begin{pmatrix} 1 + z^2 & -2z & 0 \\ -2z & 1 + z^2 & 0 \\ 0 & 0 & 1 - z^2 \end{pmatrix} \tag{3.37}$$

The entry needed to compute the D-term of the reduced theory (3.21) is

$$\mathcal{N}_{22} = -i \tag{3.38}$$

Choosing  $n_V = 2$  gives us the freedom to gauge up to three isometries in the hypermultiplet sector. The simplest case is to gauge just one, and this is the obvious supergravity analogue of the U(1) gauging in the supersymmetric case. However in the next section it will be shown that the simplest gauging does not allow P-term potentials. In the following sections we consider theories containing one, two and three Killing vectors in turn, showing whether or not P-term potentials are allowed and considering the vacuum structure of the potential and its suitability for inflation. To find trajectories suitable for inflation we look for directions along which the matter fields are minimised but  $z$  is unconstrained. We shall call such a direction a valley for  $z$ .

### 3.2.1 Gauging one isometry

The simplest gauging is to include only one Killing vector. We start with a review of how a D-term potential can be constructed. Choose the Killing vector [13]

$$k_\Lambda = 4b_3 \frac{\partial}{\partial h} - 2(\xi b_2 - 2b_1 b_3) \frac{\partial}{\partial b_1} + 2(\xi b_1 + 2b_2 b_3) \frac{\partial}{\partial b_2} + 2(b_3^2 - e^{2h} + 1 - b_1^2 - b_2^2) \frac{\partial}{\partial b_3} \tag{3.39}$$

The moment map corresponding to this Killing vector is

$$P_{\Lambda|N=2} = \begin{pmatrix} -2e^{-h}b_1b_3 - 2b_2 + \xi e^{-h}b_2 \\ -2e^{-h}b_2b_3 + 2b_1 - \xi e^{-h}b_1 \\ -e^{-h}(b_3^2 + 1 - b_1^2 - b_2^2) - e^h + \xi \end{pmatrix} \tag{3.40}$$

The conditions for truncation (3.27) and (3.29) are satisfied on the Kähler-Hodge manifold defined by  $b_1 = b_2 = 0$ . The moment map reduces to

$$P_{\Lambda|N=1} = \begin{pmatrix} 0 \\ 0 \\ \xi - e^h - e^{-h} - e^{-h}b_3^2 \end{pmatrix} \tag{3.41}$$

Using (3.38) the D-term (3.21) becomes

$$D = \frac{2i(1 + \Phi\bar{\Phi})}{\Phi - \bar{\Phi}} - \xi \quad (3.42)$$

where  $\Phi = b_3 + ie^h$ .  $D$  contains a constant FI term.

As in the supersymmetric theory it is most straightforward to describe the P-term potential in N=2 as a rotation from the D-term case. (3.39) can be generalised by performing an inverse SO(3) rotation  $R_{ij}^{-1}(\theta, \psi, \phi)$  on the  $b_i$  fields.

$$k_{\Lambda} = 4R_{j3}b_j \frac{\partial}{\partial h} + \left( 4R_{j3}(R_{i1}R_{k1} + R_{i2}R_{k2})b_i b_j + 2\left( (R_{i3}b_i)^2 - e^{2h} + 1 \right. \right. \quad (3.43)$$

$$\left. \left. - (R_{i1}b_i)^2 - (R_{i2}b_i)^2 \right) R_{k3} + 2\tilde{\xi}(R_{i1}R_{k2} - R_{i2}R_{k1})b_i \right) \frac{\partial}{\partial b_k}$$

with corresponding momentum map

$$P_{x\Lambda|N=2} = -2R_{x3}e^h - 2\epsilon_{xyz}R_{z3}b_y - e^{-h}(2R_{y3}b_y b_x - R_{x3}b_y b_y) \quad (3.44)$$

$$- e^{-h}(1 - e^{2h})R_{x3} - e^{-h}\tilde{\xi}(R_{y1}R_{x2} - R_{y2}R_{x1})b_y$$

$$+ \tilde{\xi}\epsilon_{xyz}R_{y1}R_{z2}$$

Note that a generic rotation matrix  $R_{ij}$  gives constant terms in all three components of the moment map.

Truncation to an N=1 theory (3.27) means that either the first two components or the last component of the moment map must vanish after the truncation. Which components vanish depends on whether the index of the Killing vector lies in the retained vector or retained scalar sector. As the gauge group is Abelian this means that either the D-term or the superpotential vanishes. The two allowed cases are:

**Vanishing superpotential.** The truncation conditions  $P_{\Lambda}^1 = P_{\Lambda}^2 = 0$ ,  $k_{\Lambda}^t = 0$  require

$$\sin \theta = 0 \quad (3.45)$$

$$b_1 = b_2 = 0$$

which is the D-term case considered above with Killing vector (3.39) and reduced moment map (3.41).

The scalar potential is

$$V_D = \frac{1}{2} \left( \frac{2i(1 + \Phi\bar{\Phi})}{\Phi - \bar{\Phi}} - \xi \right)^2 \quad (3.46)$$

The potential is flat for  $z$  and so naturally gives rise to inflation. We would expect the one loop corrections to this potential to lift the flat direction so that it is gently sloped. This may also give rise to a natural end for inflation as in the hybrid inflationary scenario discussed in section 2.1. The supersymmetric minimum of this potential is at

$$2i(1 + \Phi\bar{\Phi}) = \xi(\Phi - \bar{\Phi}) \quad (3.47)$$

The topology of the vacuum is  $S^1$  and cosmic strings can form by the Kibble mechanism, such strings were studied in [13] and were found to be BPS solutions.

**Vanishing D-term.** The truncation conditions  $P_X^3 = 0$ ,  $k_X^s = 0$  can only be satisfied if

$$\cos \theta = 0 \quad (3.48)$$

so after performing a rotation  $R_{ij}(\theta = \pi/2, \psi, \phi)$  on the  $b_i$  fields the Killing vector (3.43) is

$$\begin{aligned} k_X &= 4(\sin \psi b_1 + \cos \psi b_2) \frac{\partial}{\partial h} \\ &+ \left( 2 \sin \psi (-e^{2h} + 1 - b_3^2 + b_1^2 - b_2^2) + 2 \cos \psi (\tilde{\xi} b_3 + 2b_1 b_2) \right) \frac{\partial}{\partial b_1} \\ &+ \left( 2 \cos \psi (-e^{2h} + 1 - b_3^2 - b_1^2 + b_2^2) + 2 \sin \psi (2b_1 b_2 - \tilde{\xi} b_3) \right) \frac{\partial}{\partial b_2} \\ &+ \left( 4b_3 (\sin \psi b_1 + \cos \psi b_2) + 2\tilde{\xi} (-b_1 \cos \psi + b_2 \sin \psi) \right) \frac{\partial}{\partial b_3} \end{aligned} \quad (3.49)$$

The choice  $\theta = \pi/2$  can be made without loss of generality. Then the truncation conditions are

$$b_1 = b_2 = 0 \quad (3.50)$$

The truncated moment map is

$$P_{X|N=1} = \begin{pmatrix} \sin \psi \\ \cos \psi \\ 0 \end{pmatrix} (e^{-h}(b_3^2 - 1) - e^h + \tilde{\xi}) + \begin{pmatrix} \cos \psi \\ -\sin \psi \\ 0 \end{pmatrix} b_3 (2 - e^{-h} \tilde{\xi}) \quad (3.51)$$

To include  $z$ , the inflaton field, in the superpotential we choose the Killing vector (3.49) to be  $k_1$ , then with the Kähler potential (3.36) and setting  $\Phi = b_3 + ie^h$  (3.19) becomes

$$L = \frac{ize^{i\psi_1}}{\sqrt{2(1-z\bar{z})}} \left( \frac{\Phi^2 - 1 - i\tilde{\xi}\Phi}{\Phi - \bar{\Phi}} \right) \quad (3.52)$$

which is composed of two parts; the total Kähler potential

$$\begin{aligned} K &= K_V + K_Q \\ &= -\ln(2(1 - \bar{z}z)) - 2\ln(-i(\Phi - \bar{\Phi})) \end{aligned} \quad (3.53)$$

and the superpotential

$$W = ze^{i\psi_1} (\Phi^2 - 1 - i\tilde{\xi}\Phi) \quad (3.54)$$

The reduced scalar potential, (3.31), is

$$V_F = \frac{2(1 - 3z\bar{z})}{1 - z\bar{z}} \left| \frac{\Phi^2 - 1 - i\tilde{\xi}\Phi}{\Phi - \bar{\Phi}} \right|^2 + \frac{z\bar{z}}{1 - z\bar{z}} \left| \frac{2 - 2\Phi\bar{\Phi} + i\tilde{\xi}(\Phi + \bar{\Phi})}{\Phi - \bar{\Phi}} \right|^2 \quad (3.55)$$

which is an F-term potential.

To see if this potential can give rise to inflation notice that the potential can be extremized in the  $\Phi$  directions by setting  $\Phi = i$  ( $b_3 = h = 0$ ). This is a minimum if both the following conditions hold:

$$z\bar{z}(6 - \tilde{\xi}) > 2 - \tilde{\xi} \quad (3.56)$$

$$z\bar{z}(3\tilde{\xi} - 2) > \tilde{\xi} - 2 \quad (3.57)$$

Along this trajectory the potential is

$$V(\Phi = i) = \frac{(\tilde{\xi} - 2)^2(1 - 3z\bar{z})}{2(1 - z\bar{z})} \quad (3.58)$$

If  $\tilde{\xi} \neq 2$  the fields are not always confined to this trajectory so for inflation to occur the initial conditions must be chosen carefully. Even so the potential rapidly becomes steeply sloped violating the slow roll conditions. If  $\tilde{\xi} = 2$ , then  $\Phi = i$  is a flat direction that minimises the potential; inflation can occur as  $z$  rolls along this valley.

The supersymmetric minimum of the potential occurs at  $\nabla_z L = \nabla_\Phi L = 0$  which requires

$$\begin{aligned} z &= 0 \\ \Phi^2 - 1 - i\tilde{\xi}\Phi &= 0 \end{aligned} \quad (3.59)$$

So the supersymmetric vacuum is unique if  $\tilde{\xi} = 2$  and no topological defects form. If  $\tilde{\xi} \neq 2$  then there are two possible supersymmetric vacua which may give rise to domain walls.

With a simple gauging it is possible to construct a triplet of moment maps containing constant terms in N=2 supergravity (3.44). However subsequent truncation to N=1 supergravity means that the potential is either an F-term potential (3.55) or a D-term potential (3.46). Combinations of the two types which we would have expected in a P-term model are no longer allowed.

### 3.2.2 Gauging two isometries

With a simple gauging the supergravity truncation conditions allow us to have either an F-term potential or a D-term potential but not a combination of the two. However by gauging more than one symmetry it is possible to get FI terms in the D-term and superpotential of the theory. The simplest change we can make to the gauge group is to include two Killing vectors instead of just one. We choose  $k_1$  to take the form (3.49) in order to generate an F-term and  $k_2$  to take the form (3.39) in order to generate a D-term after truncation. We relabel  $\tilde{\xi} = \xi_1$  and  $\xi = \xi_2$  so that the origin of each FI term is clear. The Killing vectors must satisfy

$$[k_\Delta, k_\Sigma] = f_{\Delta\Sigma}^\Gamma k_\Gamma \quad (3.60)$$

where  $f_{\Delta\Sigma}^\Gamma$  are the structure constants of the gauge group. This requires

$$\xi_1 = -\xi_2 = \pm 2 \quad (3.61)$$

$$f_{12}^1 = f_{21}^1 = f_{12}^2 = f_{21}^2 = 0 \quad (3.62)$$

which gives an Abelian gauge group.

A consistent truncation can be achieved by setting  $b_1 = b_2 = 0$  and  $L^2 = 0$  and reducing the geometry to that considered in the previous section. Notice that after truncation only  $k_2$  lies in the reduced Kähler-Hodge manifold, so the reduced N=1 theory has a U(1)

gauging. The reduced potential is constructed from (3.38), (3.42) and (3.52)

$$V = \frac{2(1 - 3z\bar{z})}{1 - z\bar{z}} \left| \frac{\Phi^2 - 1 - i\xi_1\Phi}{\Phi - \bar{\Phi}} \right|^2 + \frac{z\bar{z}}{1 - z\bar{z}} \left| \frac{2 - 2\Phi\bar{\Phi} + i\xi_1(\Phi + \bar{\Phi})}{\Phi - \bar{\Phi}} \right|^2 + \frac{1}{2} \left( \frac{2i(1 + \Phi\bar{\Phi})}{\Phi - \bar{\Phi}} - \xi_2 \right)^2 \tag{3.63}$$

where the FI terms are constrained by (3.61).

To look for inflation in this potential notice that  $\Phi = i$  is an extremum in the  $\Phi$  directions. Along this direction

$$V(\Phi = i) = \begin{cases} \frac{8(1-3z\bar{z})}{1-z\bar{z}} & \text{if } \xi_1 = -2, \xi_2 = 2 \\ 8 & \text{if } \xi_1 = 2, \xi_2 = -2 \end{cases} \tag{3.64}$$

If  $\xi_1 = -2, \xi_2 = 2$  then  $\Phi = i$  is only a minimum if  $z\bar{z} < 1/2$  and the potential rapidly becomes steeply sloped once the fields move away from the maximum at  $z = 0$ . It seems unlikely that sufficient efolds of inflation could occur in such a potential. If  $\xi_1 = 2, \xi_2 = -2$  then  $\Phi = i$  is a global minimum and inflation would occur as  $z$  rolls along this flat direction.

A supersymmetric minimum requires  $\nabla_{\Phi}L = \nabla_zL = D = 0$  but because  $\xi_1$  and  $\xi_2$  are not equal there is no solution to these equations. There is no supersymmetric minimum in this system.

### 3.2.3 Gauging three isometries

The simple geometry we have chosen allows us to gauge up to three isometries. Including three Killing vectors of the form we have been considering gives a P-term potential which behaves differently to that of the previous section. We take one Killing vector  $k_2$  of the form (3.39) to generate the D-term and two linearly independent vectors  $k_0, k_1$  each of the form (3.49) to generate the F-term. For  $k_0, k_1$  to be linearly independent we must have  $\sin(\psi_0 - \psi_1) \neq 0$ , where  $\psi_i$  is the Euler angle appearing in the expression for  $k_i$  (3.49). For these Killing vectors to generate a group, we must impose

$$\xi_0 = \xi_1 = \xi_2 = 2 \tag{3.65}$$

and the structure constants are

$$f_{01}^0 = f_{01}^1 = f_{21}^2 = f_{20}^2 = 0 \tag{3.66}$$

$$f_{20}^1 = -f_{21}^0 = \frac{8}{\sin(\psi_1 - \psi_0)} \tag{3.67}$$

$$f_{21}^1 = -f_{20}^0 = \frac{8 \cos(\psi_1 - \psi_0)}{\sin(\psi_1 - \psi_0)} \tag{3.68}$$

$$f_{01}^2 = 8 \sin(\psi_1 - \psi_0) \tag{3.69}$$

and their permutations.

A consistent truncation requires  $b_1 = b_2 = 0$  and  $L^2 = 0$ . Notice that after the truncation only  $k_2$  lies in the reduced Kähler-Hodge manifold, so the N=1 gauge group is

U(1). However, as  $P_\Sigma^0$  no longer vanishes, the potential contains relics of the original N=2 non-Abelian gauge group. The moment maps are computed from  $k_0, k_1, k_2$  as before and

$$P_2^0 = \frac{8i \sin(\psi_1 - \psi_0)(z - \bar{z})}{1 - z\bar{z}} \quad (3.70)$$

which gives

$$L = \frac{i(e^{i\psi_0} + ze^{i\psi_1})}{\sqrt{2(1 - z\bar{z})}} \left( \frac{\Phi^2 - 1 - 2i\Phi}{\Phi - \bar{\Phi}} \right) \quad (3.71)$$

and

$$D = \frac{2i(1 + \Phi\bar{\Phi})}{\Phi - \bar{\Phi}} - 2 + \frac{8i \sin(\psi_1 - \psi_0)(z - \bar{z})}{1 - z\bar{z}} \quad (3.72)$$

The reduced scalar potential is

$$V = \frac{8|1 + ze^{i(\psi_1 - \psi_0)}|^2}{|1 - z\bar{z}|} \left( \frac{i(1 + \Phi\bar{\Phi}) - (\Phi - \bar{\Phi})}{\Phi - \bar{\Phi}} \right) + \frac{1}{2} \left( \frac{2i(1 + \Phi\bar{\Phi})}{\Phi - \bar{\Phi}} - 2 + \frac{8i \sin(\psi_1 - \psi_0)(z - \bar{z})}{1 - z\bar{z}} \right)^2 \quad (3.73)$$

$\Phi = i$  extremizes the potential in the  $\Phi$  directions and is a minimum if

$$z\bar{z} + i(2 \sin(\psi_1 - \psi_0) + 1)(z - \bar{z}) + 1 > 0 \quad (3.74)$$

The potential along this direction is

$$V(\Phi = i) = \frac{32 \sin^2(\psi_1 - \psi_0)|z - \bar{z}|^2}{(1 - z\bar{z})^2} \quad (3.75)$$

We can tune the slope of the potential so that inflation can occur in this valley. The fields will eventually settle into the minimum at  $z = 0$ . The conditions for a supersymmetric minimum  $\nabla_z L = 0, \nabla_\Phi L = 0$  and  $D = 0$  are satisfied when

$$\Phi = i, \quad z = \bar{z} \quad (3.76)$$

so the minimum reached at the end of inflation is supersymmetric.

#### 4. Conclusion

The study of P-term scalar potentials in supergravity is interesting because the P-term model is the effective theory of the D3/D7 brane inflation model, one of the most promising models of string theory inflation. P-term models have been studied in supersymmetry and in rigid supergravity but there were thought to be problems involved in lifting the model to supergravity. Previously the P-term model has been studied in the rigid limit of supergravity where all P-term potentials can inflate and with some tuning of the parameters can produce sufficient e-folds of inflation and density perturbations of the correct size to agree with observations. Cosmic strings are formed at the end of inflation.

The major obstacle to constructing a P-term theory in supergravity was that it was not known how to include FI terms in N=2 supergravity. We have shown that FI terms



can be produced in both the superpotential and the D-term of the theory by considering the truncation to an N=1 theory. If only one isometry is gauged this truncation restricts the form of the N=1 scalar potential so that it contains either F-terms or D-terms but not both. It is not possible to construct a P-term model with a simple gauging by this method. If more than one isometry is gauged the potential can include F-terms and D-terms both containing FI constants. Although the gauge group of the N=2 theory is now more complicated the gauge group of the reduced N=1 theory is always the simple Abelian case where only one isometry is gauged. Therefore we believe these models are the supergravity analogues of the well known supersymmetry P-term potentials.

One of the nice properties of the supersymmetric theory was that by rotating the vector of FI terms it was possible to move from a D-term potential to an F-term one, and the magnitude of the FI vector was undetermined. This is no longer true in supergravity, where after the truncation to an N=1 theory the position of the FI terms is fixed. If we gauge more than one symmetry the size of the FI terms is also fixed. In each of the potentials we have considered we have demonstrated a direction which could give rise to inflation. It is a subject for further research whether these potentials give rise to sufficient e-folds of inflation, and whether they match cosmological observations. This is in progress. In these classes of models cosmic strings are not formed at the end of inflation. This could alleviate the parameter constraints on such models.

### Acknowledgments

We would like to thank P. Brax and M. Esole for useful discussions, and A. Achucarro, A. Van Proeyen, R. Kallosh and K. Sousa for clarifying comments. This work is supported in part by PPARC. We thank CEA Saclay, the Galileo Galilei Institute for Theoretical Physics and the Leiden Institute of Physics for their hospitality and the INFN and the European Science Foundation (ESF) for partial support during the completion of this work.

### A. Euler angles

We use the following parameterisations for an SO(3) rotation in terms of Cayley-Klein parameters

$$R = \begin{pmatrix} \frac{1}{2}(a^2 - b^{*2} + a^{*2} - b^2) & \frac{i}{2}(b^{*2} - a^2 + a^{*2} - b^2) & -ab - a^*b^* \\ \frac{i}{2}(a^2 + b^{*2} - a^{*2} - b^2) & \frac{1}{2}(b^{*2} + a^2 + a^{*2} + b^2) & -i(ab - a^*b^*) \\ ba^* + ab^* & i(ba^* - ab^*) & aa^* - bb^* \end{pmatrix} \quad (\text{A.1})$$

which are defined in terms of Euler angles as

$$a = e^{i(\psi+\phi)/2} \cos \frac{\theta}{2} \quad (\text{A.2})$$

$$b = ie^{i(\psi-\phi)/2} \sin \frac{\theta}{2} \quad (\text{A.3})$$

The SU(2) rotation associated with this SO(3) rotation is

$$U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \quad (\text{A.4})$$

## B. The effective potential

The effective potential in Supergravity is given by (2.9), an explicit calculation of this with the Kähler potential (2.8), superpotential (2.4) and D-term (2.5)

$$\begin{aligned}
 V = \frac{g^2}{2} e^K & \left\{ |\phi_+ \phi_-|^2 \left( 1 + \frac{|\phi_0|^4}{M_P^4} \right) + |\phi_0 \phi_-|^2 \left( 1 + \frac{|\phi_+|^4}{M_P^4} \right) \right. \\
 & + |\phi_0 \phi_+|^2 \left( 1 + \frac{|\phi_-|^4}{M_P^4} \right) + \frac{3|\phi_0 \phi_+ \phi_-|^2}{M_P^2} \\
 & - \sin \theta \xi (e^{i\psi} \phi_+ \phi_- + e^{-i\psi} \bar{\phi}_+ \bar{\phi}_-) \\
 & \times \left( 1 + \frac{|\phi_0|^2}{M_P^2} + \frac{|\phi_0|^2}{M_P^4} (|\phi_0|^2 + |\phi_+|^2 + |\phi_-|^2) \right) + (\sin \theta \xi)^2 \\
 & \left. \times \left( 1 - \frac{|\phi_0|^2}{M_P^2} + \frac{|\phi_0|^2}{M_P^4} (|\phi_0|^2 + |\phi_+|^2 + |\phi_-|^2) \right) \right\} \\
 & + \frac{g^2}{8} (|\phi_+|^2 - |\phi_-|^2 - \cos \theta \xi)^2 \tag{B.1}
 \end{aligned}$$

## References

- [1] A.D. Linde, *Axions in inflationary cosmology*, *Phys. Lett.* **B 259** (1991) 38.
- [2] K. Dasgupta, J.P. Hsu, R. Kallosh, A. Linde and M. Zagermann, *D3/D7 brane inflation and semilocal strings*, *JHEP* **08** (2004) 030 [[hep-th/0405247](#)].
- [3] K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, *D3/D7 inflationary model and M-theory*, *Phys. Rev.* **D 65** (2002) 126002 [[hep-th/0203019](#)].
- [4] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [[hep-th/0301240](#)].
- [5] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [[hep-th/0308055](#)].
- [6] R. Kallosh, *N = 2 supersymmetry and de Sitter space*, [hep-th/0109168](#).
- [7] R. Kallosh and A. Linde, *P-term, D-term and F-term inflation*, *JCAP* **10** (2003) 008 [[hep-th/0306058](#)].
- [8] D.Z. Freedman, *Supergravity with Axial Gauge Invariance*, *Phys. Rev.* **D 15** (1977) 1173.
- [9] K.S. Stelle and P.C. West, *Relation between vector and scalar multiplets and gauge invariance in supergravity*, *Nucl. Phys.* **B 145** (1978) 175.
- [10] R. Barbieri, S. Ferrara, D.V. Nanopoulos and K.S. Stelle, *Supergravity, R invariance and spontaneous supersymmetry breaking*, *Phys. Lett.* **B 113** (1982) 219.
- [11] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology*, *Class. and Quant. Grav.* **21** (2004) 3137 [[hep-th/0402046](#)].
- [12] R. Kallosh, L. Kofman, A.D. Linde and A. Van Proeyen, *Superconformal symmetry, supergravity and cosmology*, *Class. and Quant. Grav.* **17** (2000) 4269 [[hep-th/0006179](#)].
- [13] A. Achucarro, A. Celi, M. Esole, J. Van den Bergh and A. Van Proeyen, *D-term cosmic strings from N = 2 supergravity*, *JHEP* **01** (2006) 102 [[hep-th/0511001](#)].

- [14] A. Achucarro and J. Urrestilla, *F-term strings in the Bogomolnyi limit are also BPS states*, *JHEP* **08** (2004) 050 [[hep-th/0407193](#)].
- [15] T.W.B. Kibble, *Topology of cosmic domains and strings*, *J. Phys.* **A 9** (1976) 1387.
- [16] L. Andrianopoli et al., *General matter coupled  $N = 2$  supergravity*, *Nucl. Phys.* **B 476** (1996) 397 [[hep-th/9603004](#)].
- [17] M. Esole and K. Sousa, *Half-BPS cosmic string in  $N = 2$  supergravity in the presence of a dilaton*, *JHEP* **03** (2007) 079 [[hep-th/0610124](#)].
- [18] K. Galicki, *A generalization of the momentum mapping construction for quaternionic Kahler manifolds*, *Commun. Math. Phys.* **108** (1987) 117.
- [19] L. Andrianopoli, R. D'Auria and S. Ferrara, *Consistent reduction of  $N = 2 \rightarrow N = 1$  four dimensional supergravity coupled to matter*, *Nucl. Phys.* **B 628** (2002) 387 [[hep-th/0112192](#)].
- [20] L. Andrianopoli, R. D'Auria and S. Ferrara, *Supersymmetry reduction of  $N$ -extended supergravities in four dimensions*, *JHEP* **03** (2002) 025 [[hep-th/0110277](#)].